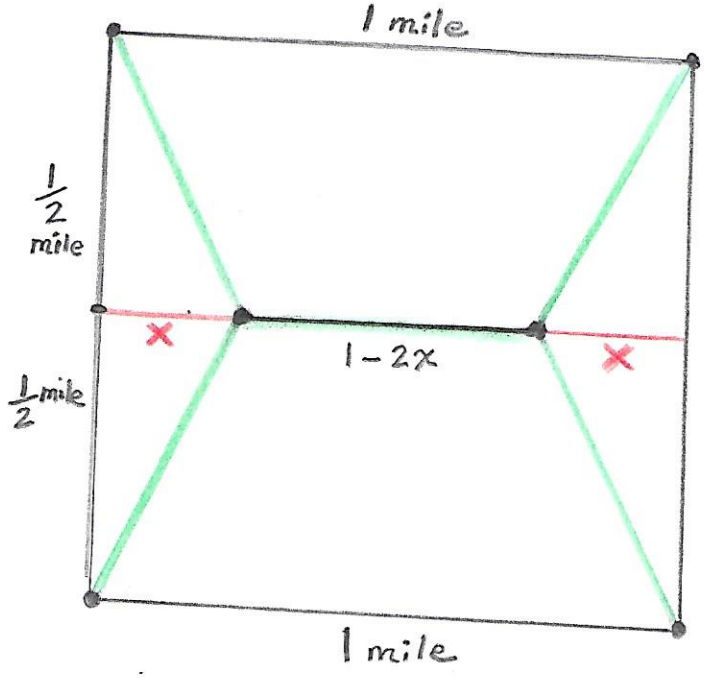
Minimum Wiring for Four Towns ON Corners of a Square Grid One Mile On a Side



$F(x)$ = Sum of lengths of line segments (green) connecting 4 towns.

$$F(x) = 4\sqrt{x^2 + \tfrac{1}{4}} + (1-2x) = 4\sqrt{\tfrac{4x^2+1}{4}} + (1-2x)$$

$$F(x) = 2(4x^2+1)^{1/2} - 2x + 1 \quad \text{then taking derivative}$$

$$F'(x) = 1(4x^2+1)^{-\frac{1}{2}} \cdot 8x - 2 \quad \text{Setting } F' \text{ equal to zero}$$

$$0 = \frac{8x}{\sqrt{4x^2+1}} - 2$$

$$\sqrt{4x^2+1} = 4x$$

$$4x^2+1 = 16x^2$$

$$1 = 12x^2$$

$$\sqrt{\tfrac{1}{12}} = x$$

Minimum $F(x)$ at $x = 1/\sqrt{12}$

$$F(x) = 4\sqrt{\tfrac{1}{3}} - \frac{2}{\sqrt{12}} + 1$$

$$F(x) = \frac{4}{\sqrt{3}} - \frac{2}{2\sqrt{3}} + 1$$

$$F(x) = \frac{3}{\sqrt{3}} + 1 = \boxed{\sqrt{3}+1}$$

minimum possible length