

Using a bunch of square roots
to change 8 to one...
or "very close" to one.



Let's put 8 on a square root diet.

$$\sqrt{8} \approx 2.828427$$

$$\sqrt{\sqrt{8}} \approx 1.681793$$

$$\sqrt{\sqrt{\sqrt{8}}} \approx 1.296839$$

$$\sqrt{\sqrt{\sqrt{\sqrt{8}}}} \approx 1.138789$$

etc.

Suppose we wish to reduce 8 all the way
to one. That would take infinitely many
square roots, which would wear out your
calculator button. But we could satisfy the
A.P. Calculus 3 decimal place accuracy.
Using the $\frac{1}{2}$ power for $\sqrt{\quad}$, and some logarithms.

$$8^{\frac{1}{N}} < 1.001$$

$$\ln 8^{\frac{1}{N}} < \ln 1.001$$

$$\frac{1}{N} \ln 8 < \ln 1.001$$

$$\frac{\ln 8}{\ln 1.001} < N$$

$$2080.48 < N \quad \text{So } N = 4096 = 2^{12}$$

is sufficient

To check our thoughts

$$8^{1/2^{12}} = \left(\left(\left(8^{\frac{1}{2}} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \dots \right)^{\frac{1}{2}} \quad \text{12 square roots} \approx 1.0005$$

That's close enough for government work!