I have been studying infinite continued fractions with two variables of the form shown to the right.

$$\cfrac{b}{a+\cfrac{b}{a+\cfrac{b}{a+\cfrac{b}{a+\cdots}}}}$$

Since the fraction contains itself, the value can be determined algebraically as follows:

$$x = \frac{b}{a + x} \Rightarrow x^2 + ax = b \Rightarrow x^2 + ax - b = 0$$

The quadratic formula gives $x = \dfrac{-a \pm \sqrt{a^2+4b}}{2}$

I noticed that certain values of a, b will give a rational result because the discriminant is a perfect square. The table below, which I worked out, gives some beautiful patterns.

(a, b)	$\sqrt{a^2+4b}$	Value of continued fraction
$(1, 2)$	$\sqrt{9}$	1
$(1, 6)$	$\sqrt{25}$	2
$(3, 4)$	$\sqrt{25}$	1
$(1, 12)$	$\sqrt{49}$	3
$(3, 10)$	$\sqrt{49}$	2
$(5, 6)$	$\sqrt{49}$	1
$(1, 20)$	$\sqrt{81}$	4

the table continues

$(3, 18)$	$\sqrt{81}$	3
$(5, 14)$	$\sqrt{81}$	2
$(7, 8)$	$\sqrt{81}$	1
$\vdots$	$\vdots$	$\vdots$

Every column has an interesting pattern!